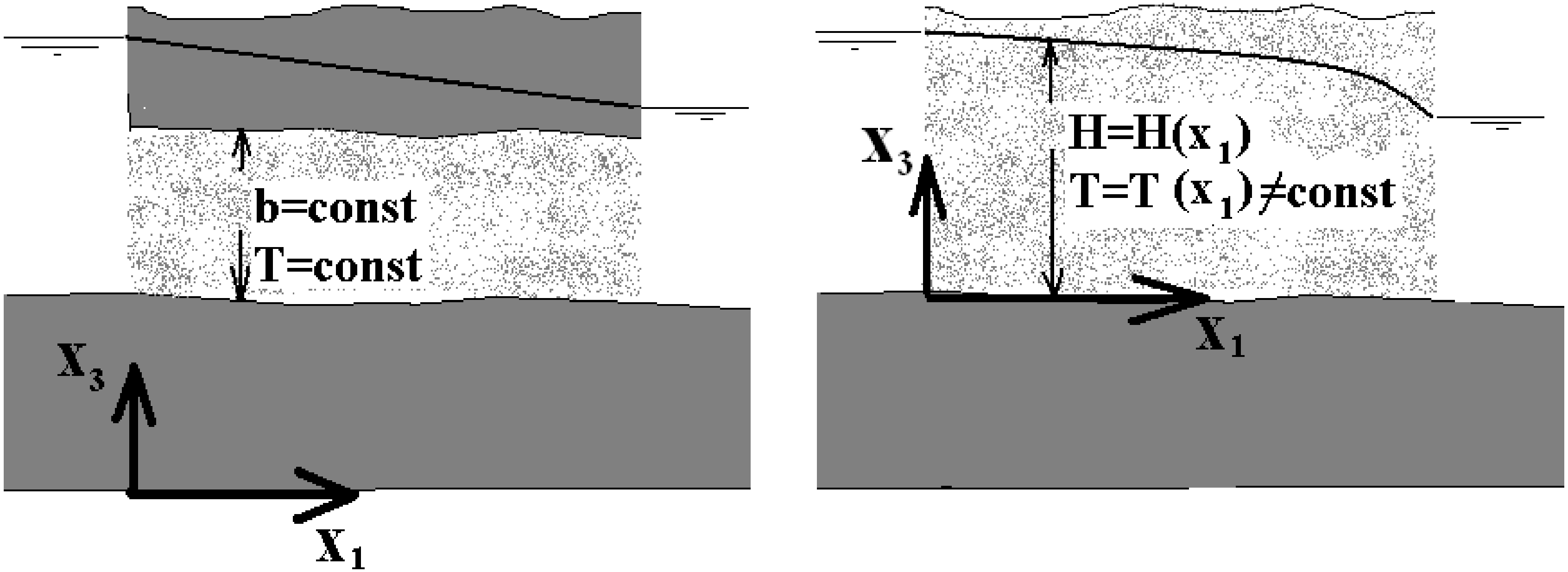
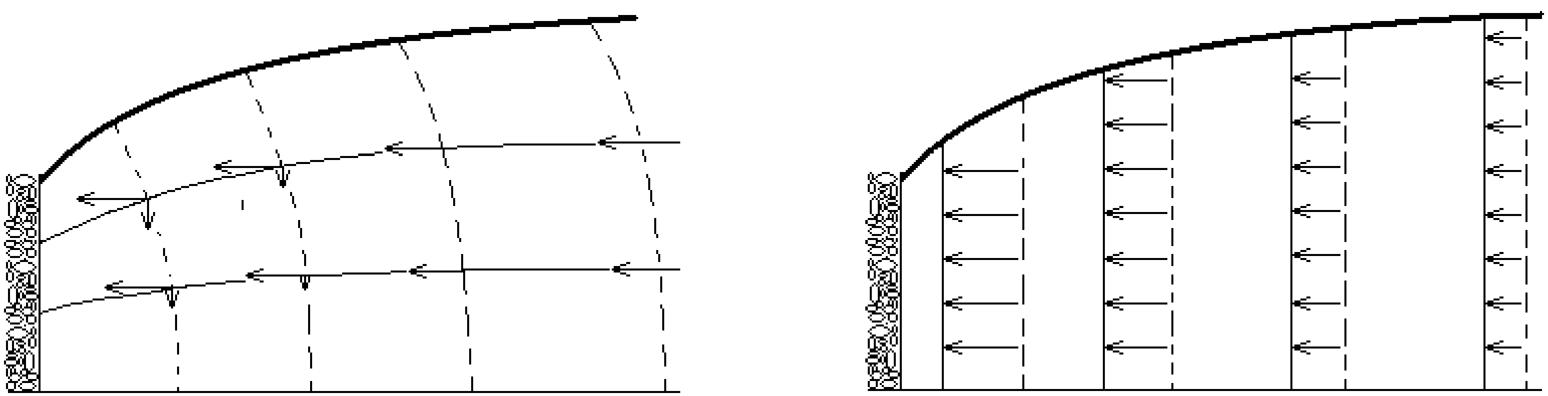
HIDRAULIKA PODZEMNIH VODA



Dipi-Forhejmerova pretpostavka



Oblici jednačine strujanja u akviferu sa slobodnom površinom

Jednačina strujanja u horizontalnoj ravni, Busineskova jednačina

- neustaljeno strujanje
- 2D, <u>ravansko</u> strujanje u osnovi, u ravni (1, 2)
- homogena, izotropna, stišljiva voda
- nehomogena, anizotropna, nestišljiva vodonosna sredina

$$e_P \frac{\partial H}{\partial t} = \frac{\partial\left(K_1 H \frac{\partial H}{\partial x_1}\right)}{\partial x_1} + \frac{\partial\left(K_2 H \frac{\partial H}{\partial x_2}\right)}{\partial x_2} + W$$

efektivna poroznost

Oblici jednačine strujanja u akviferu sa slobodnom površinom

$$\underbrace{e_P}_{\text{efektivna poroznost}} \frac{\partial H}{\partial t} = \frac{\partial \left(K_1 H \frac{\partial H}{\partial x_1} \right)}{\partial x_1} + \frac{\partial \left(K_2 H \frac{\partial H}{\partial x_2} \right)}{\partial x_2} + W$$

- ustaljeno strujanje
- sredina je homogena, izotropna sa *K*=const:

$$0 = \frac{\partial \left(H \frac{\partial H}{\partial x_1} \right)}{\partial x_1} + \frac{\partial \left(H \frac{\partial H}{\partial x_2} \right)}{\partial x_2} + \frac{W}{K}$$

Oblici jednačine strujanja u akviferu sa slobodnom površinom

Jednačina linijskog strujanja

- neustaljeno strujanje
- 1D, linijsko strujanje u pravcu (1)
- homogena, izotropna, stišljiva voda
- nehomogena, nestišljiva vodonosna sredina

$$e_P \frac{\partial H}{\partial t} = \frac{\partial \left(K_1 H \dfrac{\partial H}{\partial x_1} \right)}{\partial x_1} + W$$

anizotropija?

Oblici jednačine strujanja u akviferu sa slobodnom površinom

Jednačina linijskog strujanja

$$e_p \frac{\partial H}{\partial t} = \frac{\partial \left(K_1 H \frac{\partial H}{\partial x_1} \right)}{\partial x_1} + W$$

- ustaljeno strujanje
- $K_1 = K =$ const

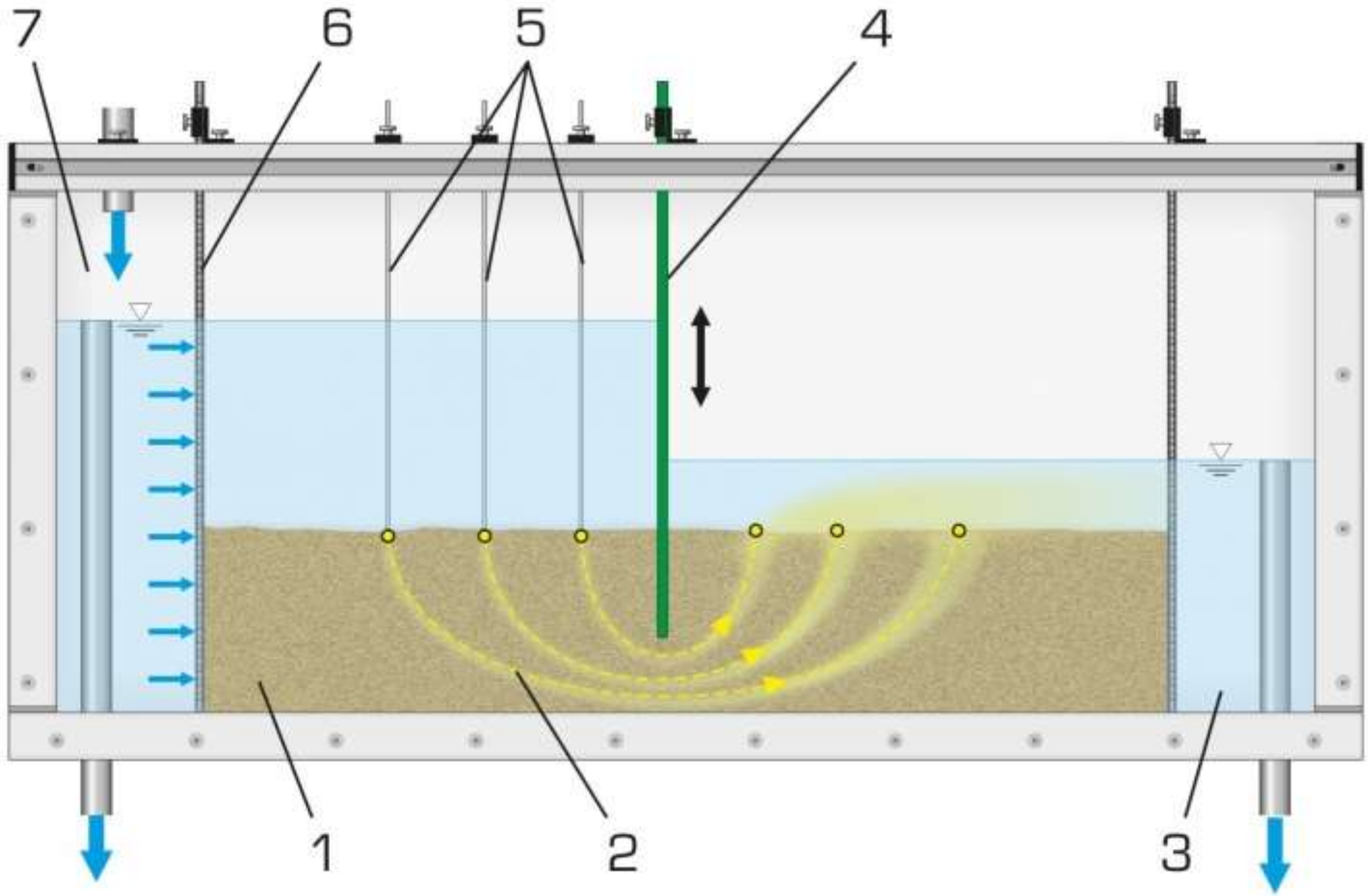
$$0 = \frac{\partial \left(H \frac{\partial H}{\partial x_1} \right)}{\partial x_1} + \frac{W}{K}$$

Problemi strujanja mogu se modelisati, simulisati različitim postupcima:

- fizički modeli („kutija sa peskom”)
- analogni modeli (viskoanalogon, elektroanalogon itd.)
- grafička konstrukcija strujne mreže
- analitički računski modeli
- numerički računski modeli.

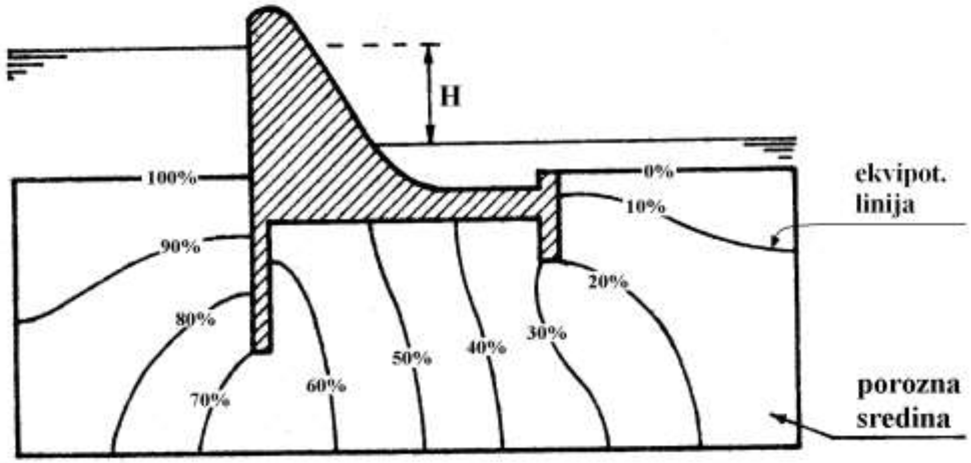
Problemi strujanja mogu se modelisati, simulisati različitim postupcima:

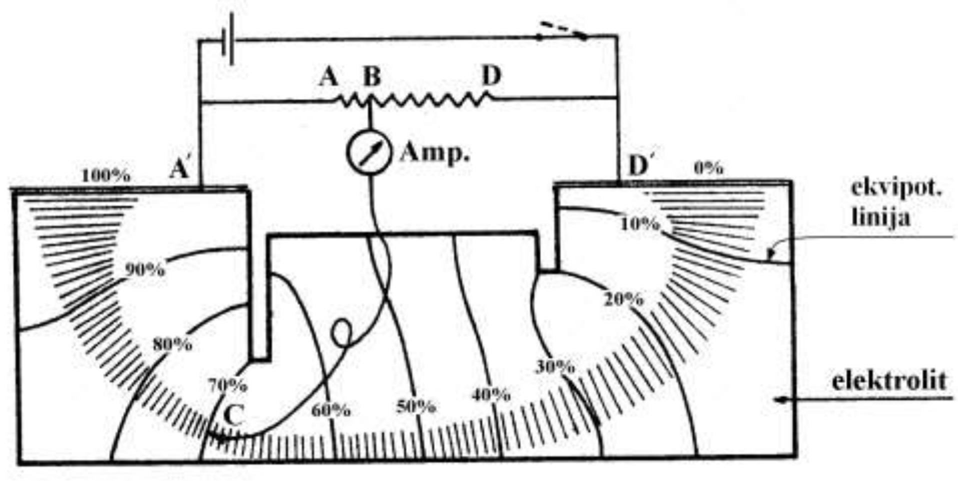
- fizički modeli („kutija sa peskom”)



Problemi strujanja mogu se modelisati, simulisati različitim postupcima:

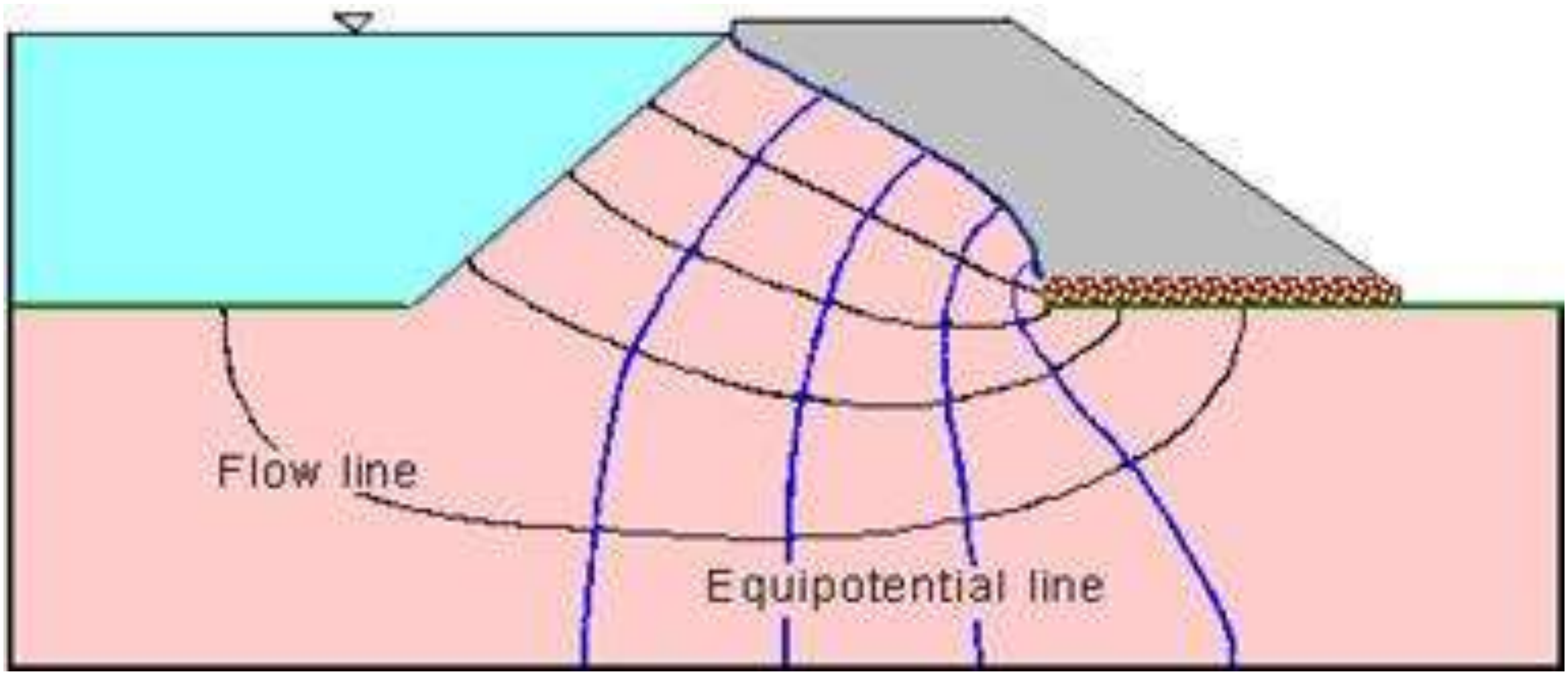
- analogni modeli (viskoanalogon, **elektroanalogon…**)





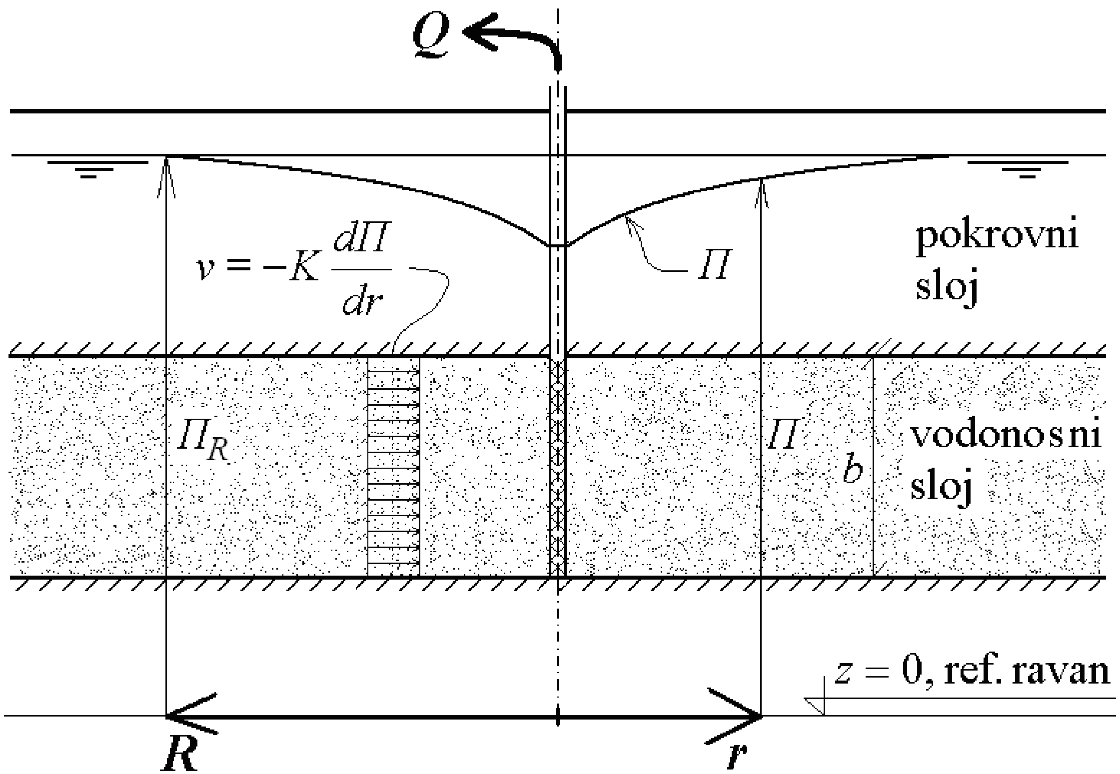
Problemi strujanja mogu se modelisati, simulisati različitim postupcima:

- grafička konstrukcija strujne mreže



Problemi strujanja mogu se modelisati, simulisati različitim postupcima:

- analitički računski modeli

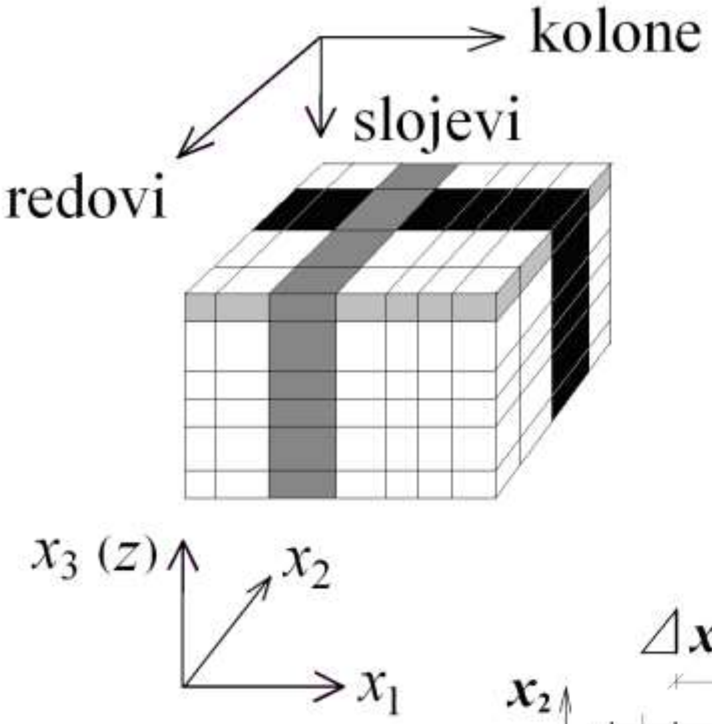


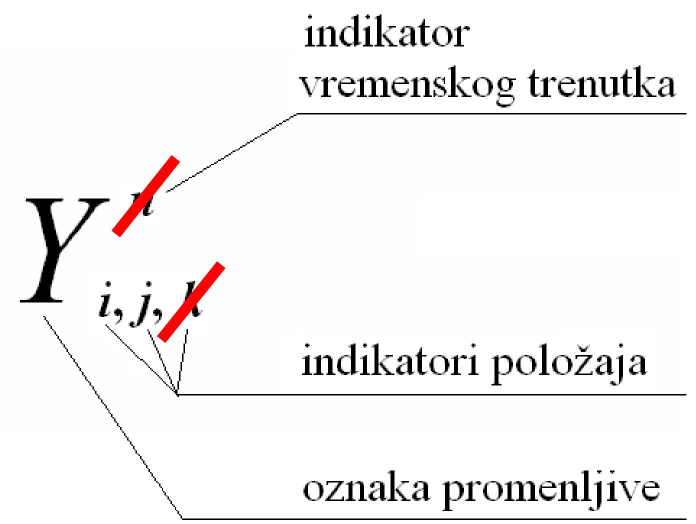
$$\Pi_2 - \Pi_1 = \frac{Q}{2Kb\pi} \ln \frac{r_2}{r_1}$$

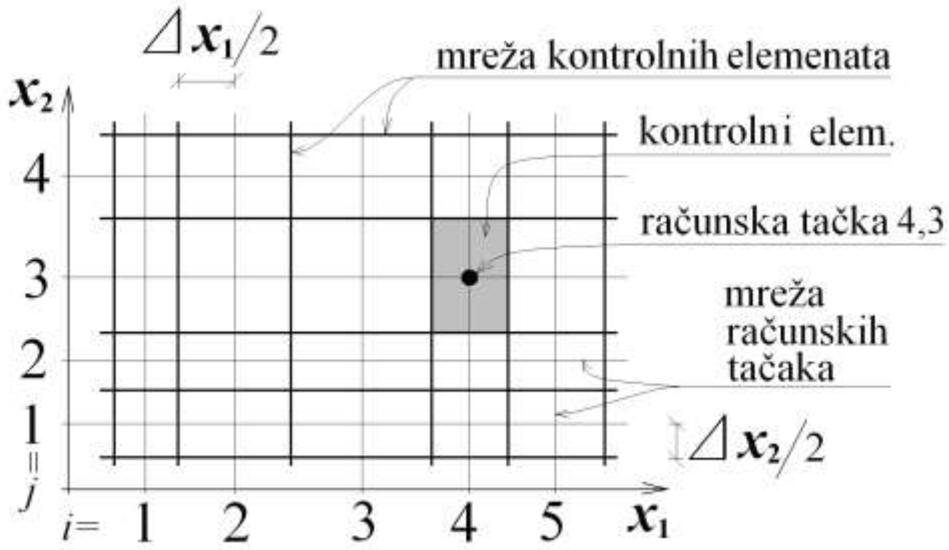
Postupak numeričkog rešavanja zadatka sastoji se od sledećih koraka:

- utvrđivanje odgovarajuće diferencijalne jednačine strujanja,
- diskretizacija jednačine (pisanje diferencijalne jednačine u vidu konačnih priraštaja),
- postavljanje računske mreže u strujnoj oblasti od interesa,
- definisanje graničnih i početnih uslova,
- primena diskretizovanih jednačina na mrežu,
- rešavanje sistema algebarskih jednačina dobijenih primenom jednačine iz tačke 2 u čvorovima iz tačke 3 za zadate uslove po tački 4.

Diskretizacija prostora i vremena – računska mreža







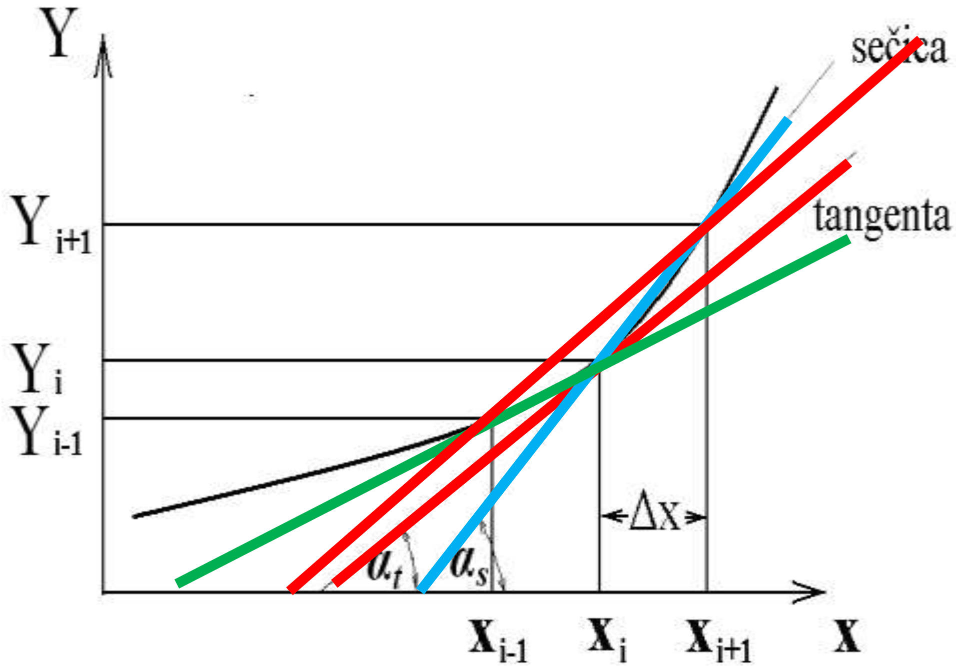
Diskretizacija izvoda i jednačine

$$\frac{dY}{dx} = \lim_{\Delta x \to 0} \frac{Y(x+\Delta x) - Y(x)}{\Delta x}$$

$$\left.\frac{\partial Y}{\partial x}\right|_{x+\Delta x/2} \approx \frac{Y_{i+1} - Y_i}{\Delta x}$$

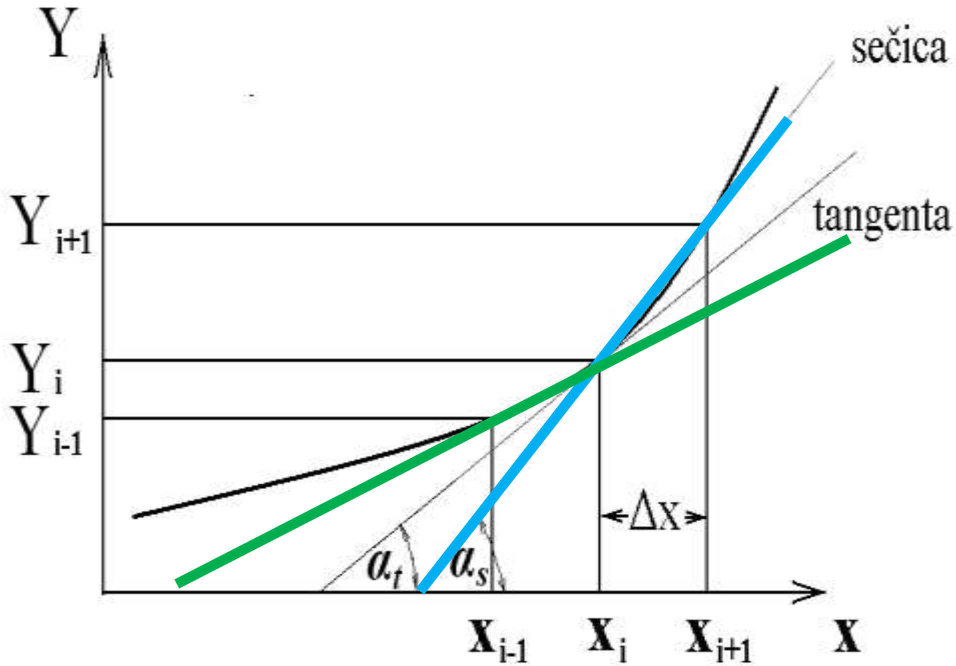
$$\left.\frac{\partial Y}{\partial x}\right|_{x-\Delta x/2} \approx \frac{Y_i - Y_{i-1}}{\Delta x}$$

$$\left.\frac{\partial Y}{\partial x}\right|_{x} \approx \frac{Y_{i+1} - Y_{i-1}}{2\,\Delta x}$$



Drugi izvod je izvod prvog izvoda:

$$\frac{\partial^2 Y}{\partial x^2} = \frac{\partial\left(\frac{\partial Y}{\partial x}\right)}{\partial x}$$



$$\left.\frac{\partial^2 Y}{\partial x^2}\right|_x \approx \frac{\left.\frac{\partial Y}{\partial x}\right|_{x+\Delta x/2} - \left.\frac{\partial Y}{\partial x}\right|_{x-\Delta x/2}}{\Delta x} = \frac{\frac{Y_{i+1}-Y_i}{\Delta x} - \frac{Y_i - Y_{i-1}}{\Delta x}}{\Delta x} = \frac{Y_{i+1} - 2Y_i + Y_{i-1}}{\Delta x^2}$$

Definisanje graničnih i početnih uslova

Naziv graničnog uslova		diskretizovano
Dirihleov	$\Pi = f$	—
Nojmanov	$\frac{\partial \Pi}{\partial n} = f$	$\frac{\partial \Pi}{\partial n} = \frac{-q_n}{K_n}$

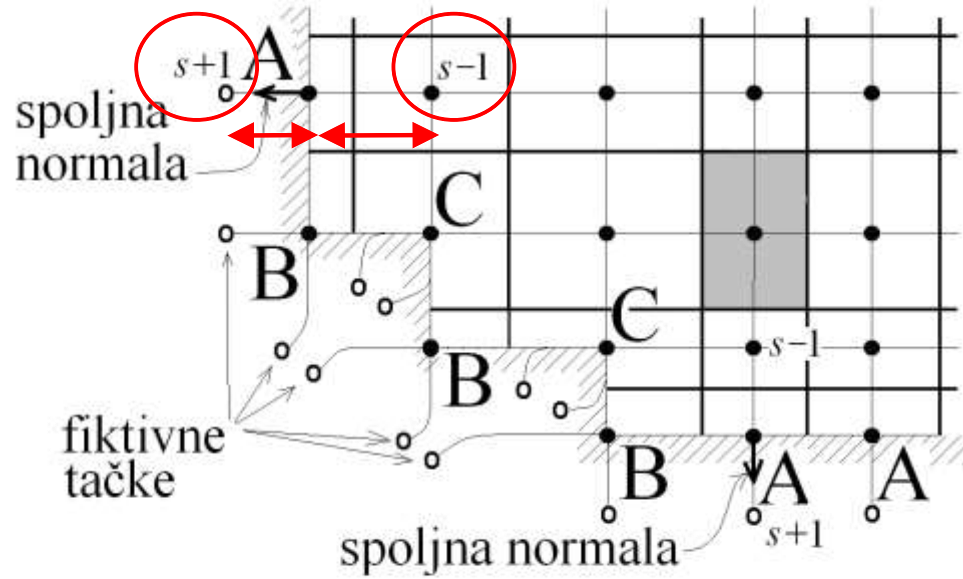
$$q_n = v_n = -K_n \frac{\partial \Pi}{\partial n}$$

Definisanje graničnih uslova u mreži računskih tačaka

$$q_n = v_n = -K_n \frac{\partial \Pi}{\partial n}$$

$$\frac{\partial \Pi}{\partial n} = \frac{-q_n}{K_n}$$

$$\frac{\Pi_{s+1} - \Pi_{s-1}}{2\Delta n} = \frac{-q_n}{K_n}$$



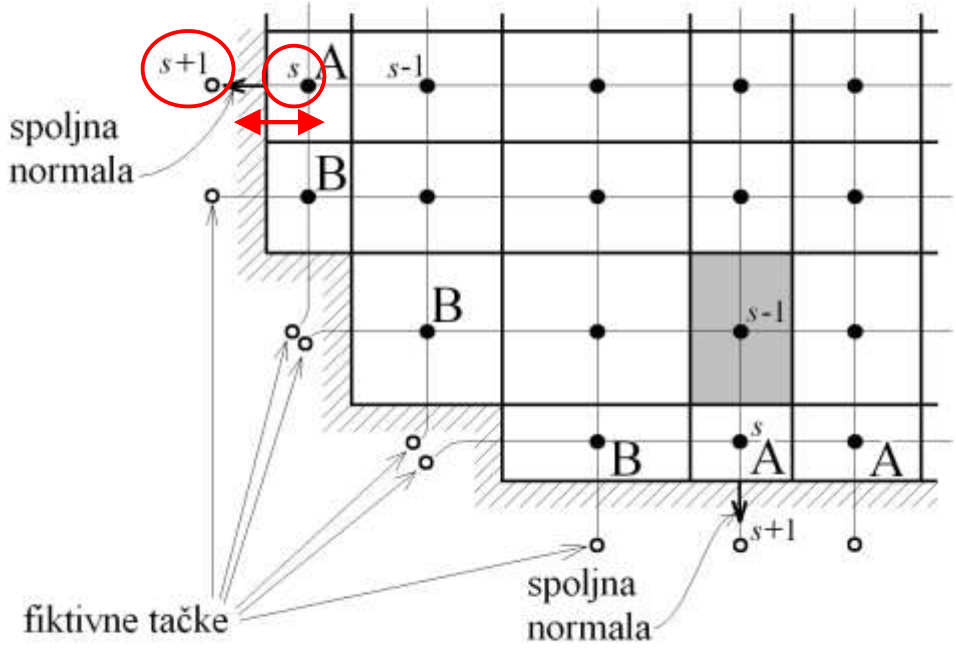
$$\Pi_{s+1} = \Pi_{s-1} - \frac{2 q_n \Delta n}{K_n}$$

Definisanje graničnih uslova u mreži kontrolnih elemenata

$$q_n = v_n = -K_n \frac{\partial \Pi}{\partial n}$$

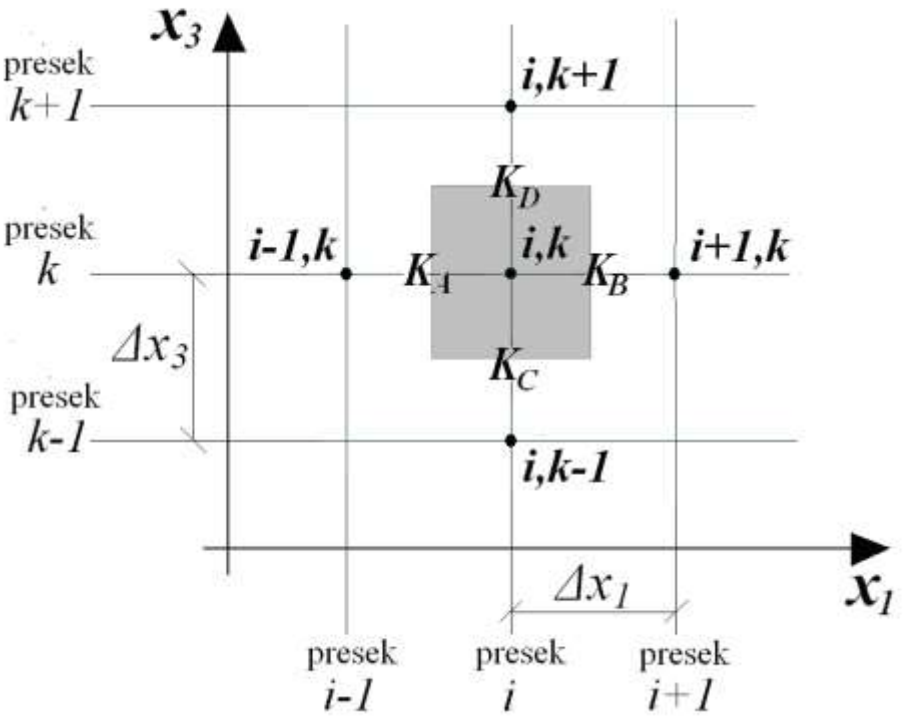
$$\frac{\partial \Pi}{\partial n} = \frac{-q_n}{K_n}$$

$$\frac{\Pi_{s+1} - \Pi_s}{\Delta n} = \frac{-q_n}{K_n}$$



$$\Pi_{s+1} = \Pi_s - \frac{q_n \Delta n}{K_n}$$

Modelisanje nehomogenosti



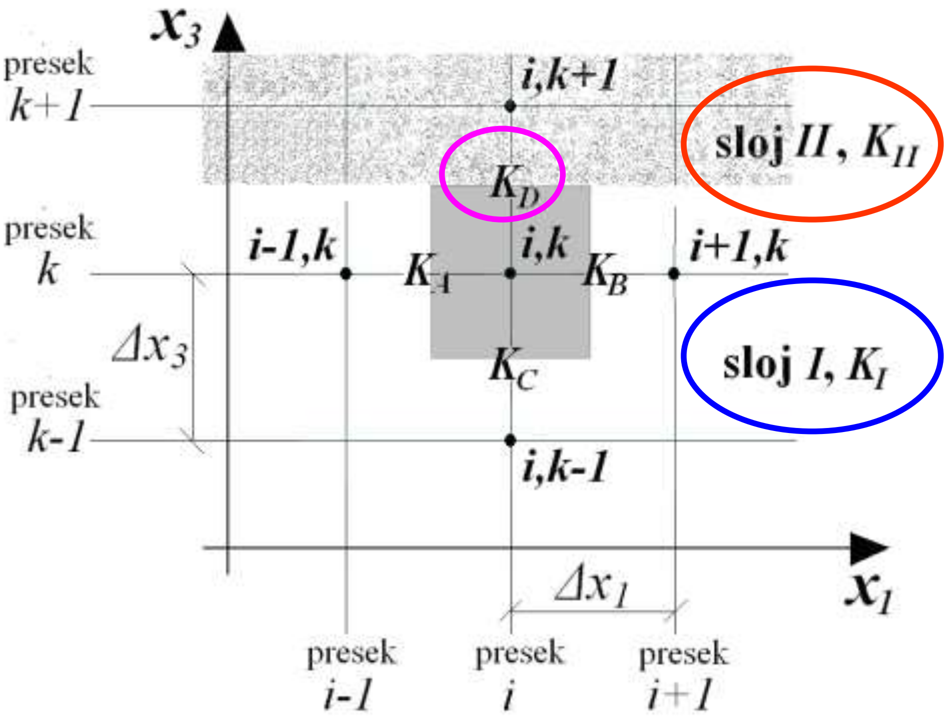
$$\frac{\partial}{\partial x_i}\left(K_i \frac{\partial \Pi}{\partial x_i}\right) = 0$$

$$\frac{\partial}{\partial x_1}\left(K_1 \frac{\partial \Pi}{\partial x_1}\right) + \frac{\partial}{\partial x_3}\left(K_3 \frac{\partial \Pi}{\partial x_3}\right) = 0$$

$$\frac{K_B \dfrac{\Pi_{i+1,k} - \Pi_{i,k}}{\Delta x_1} - K_A \dfrac{\Pi_{i,k} - \Pi_{i-1,k}}{\Delta x_1}}{\Delta x_1} + \frac{K_D \dfrac{\Pi_{i,k+1} - \Pi_{i,k}}{\Delta x_3} - K_C \dfrac{\Pi_{i,k} - \Pi_{i,k-1}}{\Delta x_3}}{\Delta x_3} = 0$$

$$\Pi_{i,k} = \frac{K_B \Pi_{i+1,k} + K_A \Pi_{i-1,k} + K_D \Pi_{i,k+1} + K_C \Pi_{i,k-1}}{K_B + K_A + K_D + K_C}$$

Modelisanje nehomogenosti

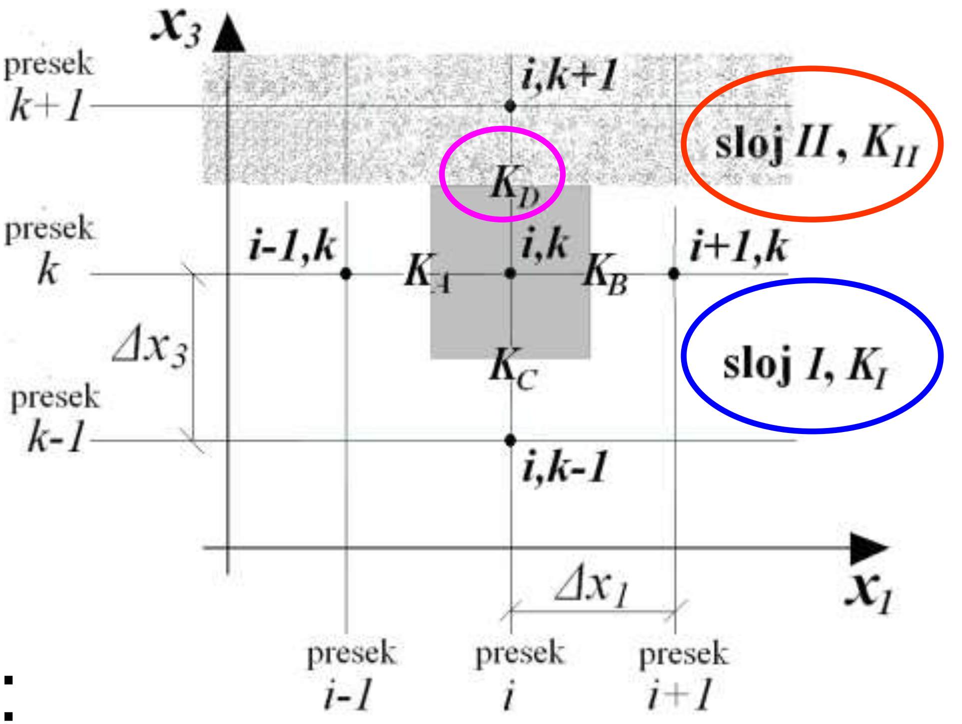


$$\Pi_{i,k} = \frac{K_B \Pi_{i+1,k} + K_A \Pi_{i-1,k} + K_D \Pi_{i,k+1} + K_C \Pi_{i,k-1}}{K_B + K_A + K_D + K_C}$$

$$K_A = K_B = K_C = K_I, \qquad K_D \neq K_{II}$$

$$\Pi_{i,k} = \frac{K_I \left(\Pi_{i+1,k} + \Pi_{i-1,k} + \Pi_{i,k-1} \right) + K_D \Pi_{i,k+1}}{3K_I + K_D}$$

Modelisanje nehomogenosti



$$K_D \neq K_{II}$$

je prenosni koeficijent:

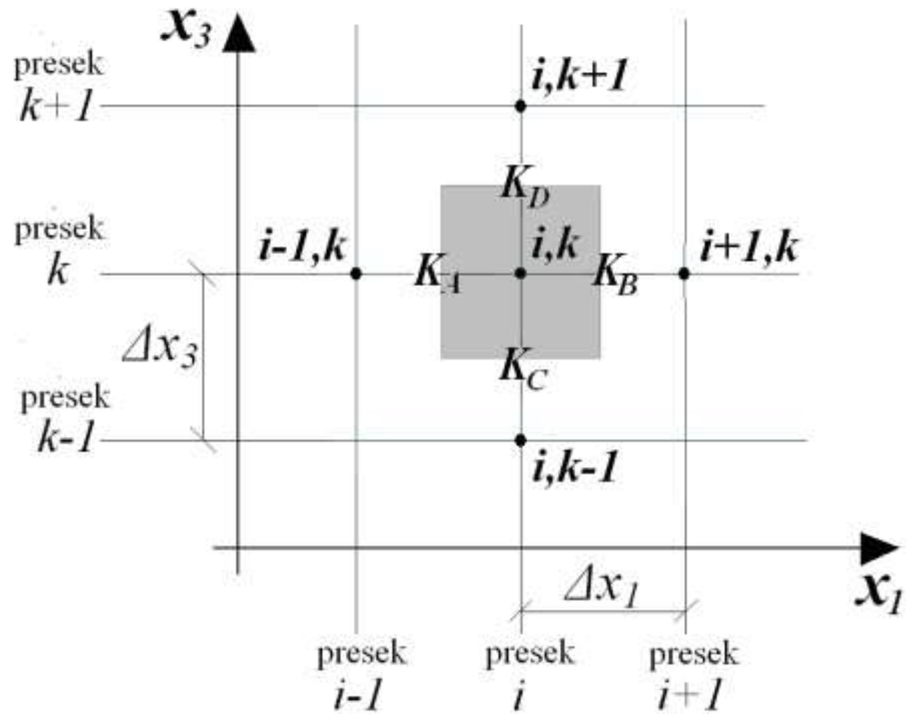
$$K_D = K_{I-II} = \left(\prod_{i=1}^{n} K_i \right)^{1/n} = \sqrt{K_I \cdot K_{II}}$$

geometrijska sredina

$$K_D = K_{I-II} = \frac{n}{\sum_{i=1}^{n} \frac{1}{K_i}} = 2 \frac{K_I \cdot K_{II}}{K_I + K_{II}}$$

harmonijska sredina

Modelisanje anizotropije



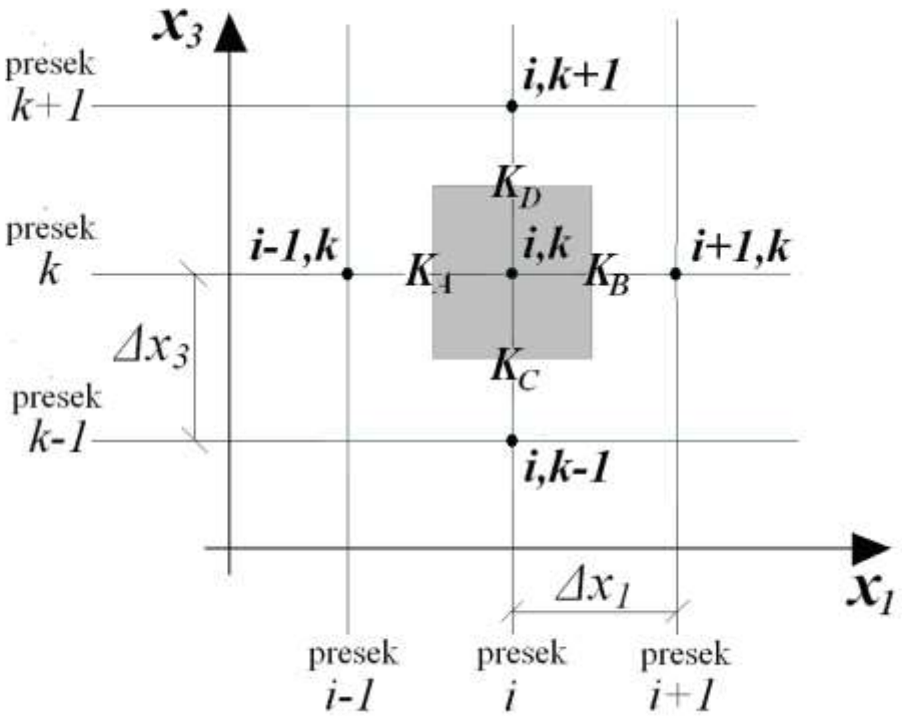
$$\frac{\partial}{\partial x_i}\left(K_i \frac{\partial \Pi}{\partial x_i} \right) = 0$$

$$\frac{\partial}{\partial x_1}\left(K_1 \frac{\partial \Pi}{\partial x_1} \right) + \frac{\partial}{\partial x_3}\left(K_3 \frac{\partial \Pi}{\partial x_3} \right) = 0$$

$$\frac{K_B \dfrac{\Pi_{i+1,k} - \Pi_{i,k}}{\Delta x_1} - K_A \dfrac{\Pi_{i,k} - \Pi_{i-1,k}}{\Delta x_1}}{\Delta x_1} + \frac{K_D \dfrac{\Pi_{i,k+1} - \Pi_{i,k}}{\Delta x_3} - K_C \dfrac{\Pi_{i,k} - \Pi_{i,k-1}}{\Delta x_3}}{\Delta x_3} = 0$$

$$\Pi_{i,k} = \frac{K_B \Pi_{i+1,k} + K_A \Pi_{i-1,k} + K_D \Pi_{i,k+1} + K_C \Pi_{i,k-1}}{K_B + K_A + K_D + K_C}$$

Modelisanje nehomogenosti



$$\Pi_{i,k} = \frac{K_B \Pi_{i+1,k} + K_A \Pi_{i-1,k} + K_D \Pi_{i,k+1} + K_C \Pi_{i,k-1}}{K_B + K_A + K_D + K_C}$$

$$K_A = K_B = K_1, \qquad K_D = K_C = K_3$$

$$\Pi_{i,k} = \frac{K_1\left(\Pi_{i+1,k} + \Pi_{i-1,k}\right) + K_3\left(\Pi_{i,k+1} + \Pi_{i,k-1}\right)}{2\left(K_1 + K_3\right)}$$

Stavljanjem da je $K_I = K_D = K$, odnosno da je

$K_1 = K_3 = K$, kako jednačina za modelisanje

nehomogene sredine:

$$\Pi_{i,k} = \frac{K_I\left(\Pi_{i+1,k} + \Pi_{i-1,k} + \Pi_{i,k-1}\right) + K_D\ \Pi_{i,k+1}}{3K_I + K_D}$$

tako i jednačina za modelisanje anizotropne sredine:

$$\Pi_{i,k} = \frac{K_1\left(\Pi_{i+1,k} + \Pi_{i-1,k}\right) + K_3\left(\Pi_{i,k+1} + \Pi_{i,k-1}\right)}{2\left(K_1 + K_3\right)}$$

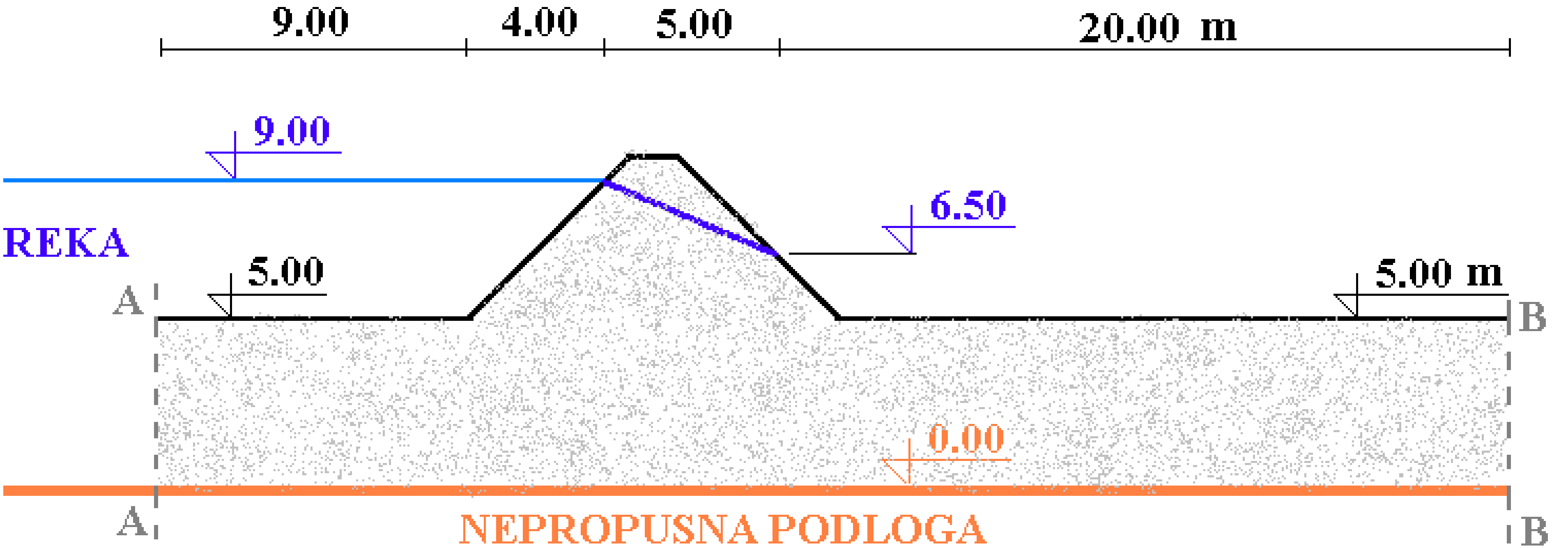
svode se na :

$$\Pi_{i,k} = \frac{1}{4}\left(\Pi_{i+1,k} + \Pi_{i-1,k} + \Pi_{i,k+1} + \Pi_{i,k-1}\right)$$

Primer modelisanja anizotropije

$$K_1 \frac{\partial^2 \Pi}{\partial x_1{}^2} + K_3 \frac{\partial^2 \Pi}{\partial x_3{}^2} = 0$$

Anizotropna sredina!



Primer modelisanja nehomogenosti (heterogene sredine)

$$\frac{\partial^2 \Pi}{\partial x_1^{\ 2}} + \frac{\partial^2 \Pi}{\partial x_3^{\ 2}} = 0$$

Heterogena sredina!

Spregnut rad tela nasipa i podloge!